The Uncertainty of Valuation
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WORKING PAPER
(NOT FOR QUOTATION. COMMENTS WELCOME)

KEYWORDS: Normal Uncertainty, Abnormal Uncertainty, Market Value, and Valuation

Abstract
Valuation is often said to be “an art not a science” but this relates to the techniques employed to calculate value not to the underlying concept itself. Valuation is the process of estimating price in the market place. Yet, such an estimation will be affected by uncertainties. Uncertainty in the comparable information available; uncertainty in the current and future market conditions and uncertainty in the specific inputs for the subject property. These input uncertainties will translate into an uncertainty with the output figure, the valuation.

The degree of the uncertainties will vary according to the level of market activity; the more active a market, the more credence will be given to the input information. In the UK at the moment the Royal Institution of Chartered Surveyors (RICS) is considering ways in which the uncertainty of the output figure, the valuation, can be conveyed to the use of the valuation, but as yet no definitive view has been taken apart from a single Guidance Note (GN5, RICS 2003) stressing the importance of recognising uncertainty in valuation but not proffering any particular solution. One of the major problems is that Valuation models (in the UK) are based upon comparable information and rely upon single inputs. They are not probability based, yet uncertainty is probability driven. In this paper, we discuss the issues underlying uncertainty in valuations and suggest a probability-based model (using Crystal Ball) to address the shortcomings of the current model.

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In those situations where a single value can be misleading it has been suggested that a range of values might be more meaningful

Gerald Brown (1991, p. 63)

Introduction
The thesis of this paper is that uncertainty is a real and universal phenomenon in valuation. The sources of uncertainty are rational and can be identified. They can be described in a practical manner and, above all, the process of identification and description of uncertainty will greatly assist many clients and will improve the content and the credibility of the valuer’s work.

The paper concentrates upon the practical the impact of uncertainty in property valuation. Uncertainty impacts upon the process in two ways; firstly the cash flows from investment are, to varying degrees, uncertain and secondly the resultant valuation figure is therefore open to uncertainty. The paper looks at how uncertainty can be accounted for in the valuation and how it can be reported to the client in an effective and meaningful way. This requires a standardized approach and we suggest that the use of a generic forecasting software package, in this case Crystal Ball, allows the valuer to work with familiar pricing models set up in Excel or Lotus 123 and to work with a predetermined set of probability distributions.

Literature Review - Risk and Uncertainty
Before we can consider uncertainty within the valuation process it is important to define what it is that we mean by uncertainty. Both within the academic literature and, more so, the property profession, the terms risk and uncertainty are often used interchangeably. Risk is seen as a euphemism for uncertainty. However, this colloquial use of the words is unhelpful in identifying the principal issues involved. It is important to define these words more precisely.

Definitions and discussion about risk and uncertainty are the cornerstone of a number of papers and books (see for example Bryne, 1995, Hargitay and Yu, 1993; Pellat, 1972; Pyhrr, 1973; Robinson, 1987; Sykes, 1983; Whipple, 1988; Wooford, 1978). The definitions that we are adopting follow the work of Byrne and Cadman (1984);

1 An alternative would be to use @risk which is a very similar software package
Uncertainty: is anything that is not known about the outcome of a venture at the time when the decision is made.

Risk: is the measurement of a loss identified as a possible outcome of the decision.

It is generally agreed that uncertainty is due to the lack of knowledge and poor or imperfect information about all the inputs that can be used in the analysis. In the context of valuation this refers to the input variables; the comparable information. If we are unable to confirm the veracity of the inputs then the resulting outcomes (valuation) is partially uncertain. However, if we are able to assign a probability to the input variables it will allow us to determine the range of possible outcomes. The output is therefore a measure of risk (Byrne, 1995).

The outcome of a valuation is only certain if we can accurately predict the future. Given that is not possible, there will always be an element of risk that the “actual” value differs from the predicted estimate. With a single point valuation, a single figure is produced with no understanding of the uncertainty pertaining to the input variables and thus no measure of the resulting risk. An improvement on this method would be to undertake the same valuation a limited number of times, allowing the user to change the input variables and recalculate each time to derive a number of possible outcomes or values. This analysis is a simple sensitivity or scenario analysis but is restricted to (maybe) only three or four scenarios based on an subjective assessment of how the input variables should be changed. A more robust model would allow the user to simulate a much larger range of possible outcomes.

**Literature Review - Simulation**

Probability theory is a way of measuring uncertainty (Cadman & Byrne, 1994). It allows the user to identify a range of outcomes for the most important variables and to assign probabilities to these variables. Simulation is a further development of probability analysis and Monte Carlo simulation has been an important component of quantitative risk since 1960s (see Hertz, 1964). The underlying premise of Monte Carlo simulation is to carry out the process, in this case valuation, a large number of times. Instead of using a single point estimate for each input variable the user ascribes a probability distribution to each input and the Monte Carlo technique selects random numbers for each variable and produces an answer (valuation) on that basis before selecting another random input (from within the set range) and repeating the exercise. The model will carry out this process to produce a multiple of possible outcomes which can then be statistically analysed to provide an average outcome, a range, a standard deviation etc. Each output would be the distribution of the possible outcomes and the range of possible valuations figures.
The results of simulations are represented in a form of a discrete distribution (histogram) or continuum distribution (normal distribution). Those distributions allow the valuers to know about the range of the outcomes and the probability of the values at each point of the distribution (Evans, 1992). In statistics there are many forms of probability distributions, which describe both the range of the values and the likelihood of their occurrence. The normal distribution (a bell-shape distribution) is the most known and its parameters (the mean and the standard deviation) are the most used.

The variability of valuation then depends on the variability of the inputs. Therefore, the process involves the identification of these variables, defining their probability distribution and, if there is any, there correlation (or inter-relationship) and then calculate the output (valuation figure).

P. Byrne (1997), suggesting that all valuers are aware that inputs and output from appraisal and valuations are uncertain, used a technique for risk analysis (and a package called @Risk) in a Discounted Cash Flow (DCF) model to provide a better decision making model for property investments. This was echoed by Kelliher and Mahoney (2000) who used a Monte Carlo Simulation to model outcome sin the context of a long term investment decision. This was further developed by Fraser (2004) who also suggested the use of a DCF analysis to generate a number of outcomes via a simulation model.

The accuracy of the simulation depends on the quality of the data used in the models. The problem still remain the ability to specify the real range of the inputs and their probability distributions, especially for the practioners who have no familiarity with statistical measures such as mean and standard deviation. This is discussed later in the paper.

**Valuation Variance and Uncertainty**

The definition of uncertainty that is the subject of this paper is uncertainty in the individual valuation figure of the individual valuer. It is not the difference of values of the same subject property by different valuers. The observed difference between different valuer’s values is known as variance and is a very different concept to the uncertainty pertaining to the individual valuation.

The problem with variance is that information pertaining to it either has to be set up artificially with a number of valuers asked to provide valuation on a common set of properties (see Crosby et al, 1998) or the analysis relates to valuations carried out at different points of time in the market. The outcomes of such studies varies substantially and in essence simply reports that different valuers have different ideas and thus produce different valuation figures. This is a very different concept to the uncertainty pertaining to the individual valuations within the study. The former deals with uncertainty (as expressed as variance) in output, the latter is a reference to the uncertainty pertaining to
the inputs that go into the valuation to produce the specific valuation figure reported.

The UK Experience

In March 1994 the Mallinson Working Party on commercial property valuations produced its report outlining a number of initiatives that the RICS should undertake to help improve the standing of the valuation surveyor within the business world. There were 43 recommendations made by Mallinson, 42 of which have been acted upon. The remaining recommendation, recommendation 34 proposes;

**Mallinson Recommendation 34**

> Common professional standards and methods should be developed for measuring and expressing valuation uncertainty.

This recommendation was re-addressed by the RICS Carlsberg report in 2002 (see on). Similarly, French and Mallinson (1998) proffered the use of normal probability distributions in the process and argued that;

‘Normal uncertainty\(^2\) is a universal and an unsurprising fact of property valuation. The open acknowledgement of that fact, and transparent management of its implications, will enhance both the credibility and the reputation of valuers. More than that, and of even greater importance, it will enhance the utility of valuations.

There will always be a degree of uncertainty in any valuation, but it should be incumbent upon the valuer to report ‘abnormal uncertainty’. This arises when some particular condition of the market or the property leads to the valuer being unable to value with the confidence of accuracy that might normally be expected. But this paper is predominantly concerned with ‘normal uncertainty’, which is hereafter we will term only as ‘uncertainty’.

The principal problem as argued by the Mallinson Report is that that all valuations are uncertain. A valuation figure is an individual valuer’s estimate of the exchange price in the market place; it is an expert’s opinion. Despite this, clients and third parties tend to view the valuation figure as fact. Oddly, such a view does not prevail in other areas of asset valuation; all players in the stock market and, indeed the chattels and fine art market, are fully aware that the valuation is only an estimate and may not correspond with the final sale price. Yet, for real estate, there is general belief that valuations are final and exact. There is very little understanding of the uncertainty pertaining to them and that the uncertainty will vary according to market conditions and property type. Historically, the only reference to uncertainty in the RICS’ Red Book

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\(^2\) This paper is predominantly concerned with ‘normal uncertainty’, which is hereafter we will term only as ‘uncertainty’
(Appraisal and Valuation Manual - Red Book, 1996)\(^3\) is a specific reference to ‘abnormal uncertainty’.

**Uncertainty and Abnormal Uncertainty**

Abnormal uncertainty was a concept that was included in the 1996 Red Book in PS 7.5.31. (Valuation Reports). It suggested that Abnormal Uncertainty might occur when there is a significant concern about market conditions such as times of financial turmoil. Alternatively the Abnormal Uncertainty may be property specific and related to impending litigation (such as major rent review case) or in relation to the property type (maybe the building is of an unusual size).

Wherever the valuer considers that the range of uncertainty may be greater than normal then the valuer should refer in report to specific circumstances and/or lack of information, so that the client can judge the significance of the uncertainty in relation to the estimated capital value.

The odd point of this reference is that it alludes to ‘uncertainty greater than normal’, yet until the new edition of the Red Book (RICS, 2003) there was no reference in any RICS publication (apart from recommendations contained in the Carsberg Report and the Mallinson Report) to normal uncertainty. This matter was revisited in the Carsberg Report in 2002.

**The Carsberg Report**

The RICS set up the Carsberg Committee to respond to research carried out by The University of Reading and Nottingham Trent University (2001) on the impact of Client Influence on (Investment) valuations. Although the Reading/Trent research was principally concerned with how valuations influenced the workings of the (property investment) market and specifically in the fund market, Carsberg expanded the brief of his response to encompass all issues that he felt were pertinent to the interpretation and use of valuations in all circumstances.

One of the areas that he considered was the reporting of uncertainty in valuation and he made specific recommendations therewith;

**Carsberg Recommendation 15**

RICS should commission work to establish an acceptable method by which uncertainty could be expressed in a manner which will be helpful and will not confuse users of the valuation. RICS should also seek to agree with appropriate representative bodies of those commissioning and using third party valuations the circumstances and format in which the valuer would convey uncertainty.

\(^3\) Now superseded by the RICS Appraisal and Valuation Standards 2003)
This recommendation follows on directly from Mallinson and embraces the same view that uncertainty should be reported to enhance the decision making process and aide the valuation users understanding of the valuation.

It was the view of Carsberg that the RICS should commission work to establish an acceptable method of expressing the inherent level of uncertainty within a valuation. This has been embraced by the Property Valuation Forum (PVF) who has run a number of seminars to consider the market response to such a proposal. One of the outcomes of this consultative process was the introduction of Guidance Note 5 in the 2003 edition of the RICS Appraisal & Valuation Standards (see on). They have considered the ways in which uncertainty can be reported to the client and have identified three possible approaches.

1. Verbal Reporting  The valuer articulates the uncertainty pertaining to the valuation in words within the report
2. Ranking  The valuer allocates a “rank” to the valuation on a prescribed agreed basis. This may be numerical or alphabetical (i.e. 1, 2 or 3; A, B or C)
2. Statistical  The valuer conveys the uncertainty pertaining to the valuation by the use of recognised statistical information such as central tendency and/or standard deviation.

This paper will concentrate upon the third option noted but that is not suggesting that either of the other options is not appropriate. Indeed, the current RICS guidance suggests that verbal reporting is the preferred mechanism of alerting the client to the uncertainty within the valuation.

**Verbal Reporting**

In the 2003 edition of the RICS Appraisal & Valuation Standards, there is a UK Guidance note relating to uncertainty in valuation. In this guidance it states that:

*All valuations are opinions of the price that would be achieved at the valuation date. The degree of certainty will vary significantly. These variations can arise because of the inherent features of the property, the market place or the in the information available to the valuer. Where uncertainty could have a material effect on the valuation, the valuer should draw attention to this, indicating the cause of the uncertainty and the degree to which this is reflected in the reported valuation.*

Yet, contrary to the recommendations of Mallinson and Carsberg, there is no suggestion of a standard way of reporting this uncertainty to the client. By inference, the Guidance note is suggesting that the valuer reports uncertainty in valuation to the client in a form of words within the report but it does not
suggest an acceptable form of words nor any prescribed format for the measurement of the said uncertainty.

**Ranking**

A further option for reporting risk, as suggested by Mallinson (1994), which was considered by French (1995) and developed by Adair and Hutchison (2001) is to provide a simple risk score. The premise in this case is that the valuation could be provided as an indication of the risk of variance (say ‘1’ for a low risk of variation to ‘4’ for high risk of variation). The problem with this approach is that it possibly conveys a perception of “good” and “bad” valuations. When, it is not the veracity of valuation that is in question but the certainty of the specific figure. It may be a tenuous distinction, but one that could lead to significant misinterpretations in the market. If fully understood, this could be a useful and workable solution, particularly as it would be very easy to develop the ranking of individual property scores into a portfolio average. However, again for reasons noted above, this option is not considered further in this paper.

**Valuation and Market Sentiment**

The simple premise is that a valuation is a pricing model that, depending upon the implicit or explicit nature of the module used, identifies market sentiment towards pricing by a number of benchmarks (e.g. The capitalisation rate, the target rate (equated yield, market rent, market growth expectations, exit yields etc). The valuer will use the benchmark figure that he/she feels is most appropriate (most probable?) but he/she will not be 100% confident that each of the figures used is exact. There will be a degree of uncertainty pertaining to each of the inputs.

For the purposes of this paper we are seeking to identify the substance and the characteristics of the uncertainty which lies in the valuer’s mind as he or she attempts to assess the hypothetical purchaser’s view of the inputs involved. Thus we need to address the probability and range relating to the inputs not the output. A single valuation figure still needs to be provided, but an understanding of the uncertainty relating to the inputs used in the model will allow the valuer to report the uncertainty related to that specific single valuation figure.

As both Carlsberg and Mallinson suggest in their respective recommendations (15 and 34 respectively), the aim is to establish an acceptable method by which uncertainty could be expressed in a uniform and useful manner. French and Mallinson (2000) suggest that the solution must lie in the creation of some format description, accepted as a norm, which conveys the essence with simplicity, but is capable of expansion and interpretation. This would need to be presented in a prescribed professional standard, and would always be appended to a valuation figure.
In its simplest form this would be the mean expectation of value (based on the varying probability of the inputs) plus the variation pertaining to that value within that one valuation (not variance of value between different valuers). This is effectively the best estimate plus standard deviation.

French and Mallinson (2000) argued that there are six items of information that must be conveyed.

1. the single figure valuation
2. the range of the most likely observation
3. the probability of the most likely observation
4. the range of higher probability
5. the range of 100% probability
6. the skewness of probabilities

However, this is a representation of the uncertainty of the output. And the figures generated are dependant upon input benchmarks and the uncertainty relating to each of those variables. In both cases the underlying information will be represented by normal of bell distributions, skewed or otherwise. A simpler alternative may be to report the figures as a stated absolute range on a triangular basis; most probable, best and worst. This pragmatic point will be revisited once we have discussed the application of such an approach utilising probability distributions.

**Probability and Valuation**

As noted above there is a significant difference between the use of probability in looking at the range of possible outcomes between the values produced by different valuers and the range of outcomes that would be produced by an individual valuer due to the uncertainty she or he may have in the benchmarks which are utilised in the valuation model. In this paper we are concerned with the second interpretation of uncertainty. The uncertainty of the inputs.

As discussed previously, even the simplest of valuations there are likely to be a number of variables that the valuer must assess. For example, in a vacant possession office valuation, even if the office is similar to others which have been sold recently, the valuer must assess, through the eyes of the hypothetical purchaser, slight differences in location, the time since the last transaction, differences in standards of fitting, and so on. This is normally done through the use of a comparative benchmark. In the case of implicit valuations, the All Risk Yield (ARY) or the property yield. Through the use of a single yield indicator the valuer will assess the capital value of the property by a multiplier (Years Purchase or YP) derived from the yield, which is then applied to the Market Rent. In such a model, there are only two variables. The rent and the yield. However, if we assume that the initial rent has already been agreed, then the capitalisation model relies on only one variable; the yield.
The valuer will have take a view on the appropriate yield by an analysis of comparables of the sale of similar properties. Assuming that he or she analyses, say, 20 previous transactions they will have a database of 20 observed yields which will form the foundation of the valuers judgement of the appropriate yield to be applied to the subject property. This is not a mathematical exercise but a heuristic approach and the valuer’s judgement of the uncertainty pertaining to his or her final choice of yield will not be a direct correlation to the range of the observed yields. It will however be influenced by the perceived robustness of the database.

If the market is strong and there is a lot of transactional data available, it is likely that the observations will be closely aligned and that the range of the observed yields will be small. This is because available data is both more comparable to the subject property and because the transaction dates are more likely to be closer to the valuation date. However, as market conditions deteriorate, the number of direct comparables sales falls and the valuer has to rely upon observed transactions that are less comparable in terms of location, specification and time. Here the range of observed yields will be greater.

In each case the valuer will choose a yield that he or she believes is the most appropriate. It is not directly a mean of the observations, or a mode. Indeed, as the final choice of yield will be influenced by how the valuer believes that the market has moved since the transaction dates of the comparables, the final choice of yield may not be the same as any of the observations.

The process is not a science; it is a process of judgement and expert analysis. The valuer will identify the yield that he or she feels is most appropriate and use that figure to derive the rental multiplier for the valuation model. The model will produce a single answer based upon the single point estimation of the inputs.

Yet, the valuer will not be 100% certain of the input figure. In effect, they will ascribe a degree of uncertainty to their belief in the input variable being “correct”. This is a subjective probability and will vary according to the confidence level that they feel applies for that variable, in this case the ARY.

This subjective probability is currently not quantified within the model, although an expression of the valuer’s uncertainty may be articulated in market commentary accompanying the valuation. However, it would be possible to ascribe a probability distribution to this variable in accordance with the valuer’s perception of market conditions (see on).

For more complex properties the number of variables will multiply. In order to produce the valuation, the valuer must weigh all the variables, using his or her skill and experience, and decide upon the most probable conclusion for each variable that would then feed through to the final valuation figure. A more
complex pricing model with multiple variables will be considered in a later paper.

The principal valuation model used in the UK for a rack-rented or fully-let property is the income capitalisation model (sometimes referred to as the “traditional method”). This model requires the valuer (for a vacant property) to estimate the best rent readily achievable for that property and the corresponding all risk yield or capitalisation rate. The capitalisation rate is derived by the analysis of other similar sales and is effectively the initial yield (first year’s income divided by price) of the comparable properties duly adjusted to reflect the specific circumstances of the subject property. The reciprocal of the ARY will indicate the multiplier that converts Market Rent to Capital Value. For instance an ARY of 5% will give a multiplier of 20 (called the Year’s Purchase or YP) which suggest that in the market we would expect the property to sell for twenty times the rent.

This can be illustrated in Figure 1, where an office building has just been let for a Market Rental of £10,000. The valuer’s assessment of the ARY is 5% and, for the explicit model, the corresponding market target rate (or equated Yield) is 8%. This produces a capital value of £200,000.

<table>
<thead>
<tr>
<th>Implicit (Traditional) capitalisation model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Rent</td>
</tr>
<tr>
<td>YP perp  @ 5.00%</td>
</tr>
<tr>
<td><strong>Capital Value</strong></td>
</tr>
</tbody>
</table>

**Inputs and Probability Distributions**

In the previous section, it was suggested that heuristic process that the valuer would follow in the implicit model would be to assess the market for comparable sales and derive a ARY appropriate for the subject property by an intuitive interpretation of the range of yields produced by the comparables. In our example, the valuer felt that a 5% yield was appropriate for the subject property and that this choice of yield would reflect all the risks and growth potential for that property and thus would produce the appropriate value (estimate of price) in the market place at the valuation date.

However, as discussed, the valuer will have a ‘view’ on how certain he or she is about that variable and, depending upon the state of the market, how likely he or she thinks that the yield might be higher or lower. If the market is relatively
stable then the likelihood of the yield being higher than 5% should be equal to the likelihood of the yield being lower than 5%. The degree by which it might deviate from the assumed figure is again dependent upon the market conditions. If there is sufficient market evidence, the valuer will feel more certain of the market conditions and thus more confident in the ARY adopted; the corresponding range, above and below the adopted figure, will therefore be proportionally less than the range were there more uncertainty in the adopted figure.

In statistical terms this thought process can be represented by a probability distribution. Equal likelihood of the adopted figure being higher or lower would be a symmetrical distribution; an unequal probability would result in a skewed distribution. Each input into the model will be represented by a Probability Density Function (pdf), which allows us to consider a range of values instead of a single figure. The single figure is the most likely value, the uncertainty pertaining to that figure being represented by extent of the range around that figure.

**Normal Probability Distribution**

French and Mallinson suggested that the appropriate probability distribution would be a normal or bell distribution. This is a distribution that is symmetrical around a central tendency; a non-skewed distribution will have the mean, the mode and the median coinciding. In our analysis the most likely figure will be represented by the central figure (the mean) and the uncertainty by the range around that number. There is equal probability that the observed figure will be above or below the central assumed figure. The majority (99.74%) of the possible observations will lie within plus or minus three standard deviations of the mean. The standard deviation is a measure of how widely values are dispersed from the average value (the mean). The exact standard deviation will vary according to the uncertainty pertaining to the average value; the greater the uncertainty the higher the standard deviation.

However, there is a small probability that the observed figure will lie outside the three standard deviation range and, as the distribution is open ended, it is possible that the observed value will be found in the infinite tails of the normal distribution. The distribution is continuous.

Whilst the normal distribution is the most readily understood probability distribution in statistical terms, as it can be modelled with reference only to the mean and standard deviation, it does not fit closely with the heuristic processes of the market place. Obviously the valuer is happy to determine the most likely (mean) figure for the ARY but they would not think about the range either side of the mean as a percentage variation, which is the normal expression of the standard deviation. The valuer is more likely to think in terms of absolute figures either side of the mean.
Triangular Probability Distribution
This representation is much more akin to the thought process of the valuer as it requires the user to provide three absolute figures; the most likely, the maximum and the minimum. This is a closed distribution and can be symmetrical or skewed. This is a more useful tool as it information requirements mirror the likely thought process of an expert; in this case the valuer.

However, the advantage of the triangular distribution, its simplicity, is also its disadvantage. In reality the observed distributions will tend toward a normal distribution and thus by imposing a definite limit to the range, connected by a straight line relationship, it suggests that the observed values will not be concentrated around the mean and thus the outcomes are likely to have a greater spread. In statistical terms, typically the triangular distribution overestimates the variance.

Although there are other probability distributions that may be considered (e.g. Lognormal, Beta, etc), the purpose of this paper is to review approaches that might be readily acceptable by the profession. This requires the approach to be easy to implement, pragmatic and readily understood.

Applications of Probability to the Capitalisation Model
In Figure 1, we have shown the valuation of an office building at an initial agreed rent of £10,000. The valuation can be carried out implicitly to produce the capital value of £200,000.

Within the implicit capitalisation model for a “just let” building, the only uncertain variable is the ARY. By using Crystal Ball, we are able to ascribe a probability distribution to that input and, by using a Monte Carlo analysis, test the veracity of the £200,000 figure.

Monte Carlo simulation is a re-sampling iterative process. In simple terms it changes the input in the calculation by randomly choosing a figure within the defined probability distribution. It then calculates the corresponding value using that chosen input and records that value. It then repeats the process by randomly choosing another input figure. It will continue to do this until the chosen number of iterations, normally several thousand, is complete. The output is expressed as the mean of all the calculated values.

It provides a structured approach that allows the user to incorporate uncertainty into the analysis in a relatively simple form. Because each input is defined by the chosen probability density function. If there is more than one variable to be analysed, then it is possible to allow for any interrelationship between the chosen variables. For example in a DCF model, rental growth and exit yield should be negatively correlated.
The Normal Distribution – *Crystal Ball*

The capitalisation model works well when the possible inputs are normally distributed within a tight distribution. In Figure 2, we have a mean (expected ARY) of 5%. *Crystal Ball* then sets the Standard deviation as 0.5% (10% of the mean figure) and runs the Monte Carlo simulation 50,000⁴ times.

**Figure 2a – Normal Distribution; Fixed Standard Deviation**

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**Assumptions**

Assumption: capitalisation rate

Normal distribution with parameters:

- Mean: 5.00%
- Standard Dev.: 0.50%

Selected range is from -Infinity to +Infinity

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⁴ We chose 50,000 iterations as it is sufficient to allow consistent results between different simulations.
This produces the following outcome;

**Figure 2b – Output Distribution**

![Frequency Chart](image)

In numerical terms this can be represented as:

**Figure 2c – Statistical Data**

<table>
<thead>
<tr>
<th>Forecast: Capitalisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary:</td>
</tr>
<tr>
<td>Display Range is from £147,472 to £256,657</td>
</tr>
<tr>
<td>Entire Range is from £137,100 to £335,738</td>
</tr>
<tr>
<td>After 50,000 Trials, the Std. Error of the Mean is £93</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statistics:</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trials</td>
<td>50000</td>
</tr>
<tr>
<td>Mean</td>
<td>£201,973</td>
</tr>
<tr>
<td>Median</td>
<td>£199,944</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>£20,871</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Here it can be seen that the expected mean (capital value) of £201,973 is not significantly different from the £200,000 produced by the discreet use of the implicit model. But the advantage of the Monte Carlo simulation (using **Crystal Ball**) is that provides additional information about the certainty of the result. In this case, the standard deviation (of £20,871) is a representation of the uncertainty. The skewness (of 0.63) represents the degree of asymmetry of the distribution around its mean. Here the positive skewness indicates a distribution with an asymmetric tail extending toward more positive values. Whereas a negative skewness would indicate a distribution with an asymmetric tail extending toward more negative values.
However, the normal distribution has the pragmatic drawback that the user is unlikely to express their view of uncertainty as a standard deviation. Instead, as previously discussed, it is likely that they would suggest an absolute range.

**The Triangular Distribution – *Crystal Ball***

Although the capitalisation model works best with the assumption of a normal distribution, it is more pragmatic that the choice of model should be driven by the ease of articulating the uncertainty. A valuer is likely to say that the expected ARY is 5%, although it may be as low as 4.5% or as high as 5.5%. This can be directly inputted into *Crystal Ball* as a most likely, maximum and minimum and again run for 50,000 simulations.

**Assumptions**

Assumption: capitalisation rate

Triangular distribution with parameters:
- Minimum 4.50%
- Likeliest 5.00%
- Maximum 5.50%

Selected range is from 4.50% to 5.50%

**Figure 3a – Triangular Distribution; Likeliest, Maximum and Minimum**

**Figure 3b – Output Distribution**
Again the observed mean of £200,369 is not dissimilar from the original result of £200,000. However the standard deviation is lower at £8,181 as the input range was curtailed between 4.5% and 5.5%.

Applications of Probability to a second related variable

As shown in Figure 1, the implicit capitalisation model will produce a capital value of £200,000. However, this assumed that the property had just been let and the rent had been fixed. As noted earlier, it is possible when undertaking the valuation of a vacant property that the valuer will have to estimate the market rent in addition to the capitalisation rate. We can therefore extend the model to incorporate the extra variable of rent. However, as noted, these inputs are not independent and thus it is necessary not only to consider the range of uncertainty but the inter-relationship of the two variables, rent and ARY. This is illustrated in Figure 4. Here we have decided to apply the triangular distribution. Although it is recognised that the normal distribution is more statistically robust, the Triangular representation is much more akin to the thought process of the valuer as it requires the user to provide three absolute figures: the most likely, the maximum and the minimum for each input.

The Triangular Distribution – Crystal Ball

In this example the valuer is required to identify the ARY and the corresponding market rental. In this case, the ARY was estimated to be 5%, a low of 4.5% and a high of 5.5%. Correspondingly the rental was a low of £9,500, a high of £10,500 and a most likely of £10,000. However as the two variables are interrelated, it has been necessary to add in a correlation factor between the two variables. The relationship between rent and ARY is a negative relationship. As rents increase in the market we would expect the ARY to fall. This is because as rents start increasing, the market perception will be that property is a more attractive asset than
it was and thus the multiplier (YP) against rent will increase. As the YP is the reciprocal of the ARY, then the ARY decreases as property becomes more attractive and hence the negative correlation. However, the correlation is not 100%. That is, as the rent increase by (say) 10%, the ARY does fall by the same percentage rate. It would be possible to analyse past data to derive a robust correlation coefficient but indicatively a correlation of -0.32 (i.e. the ARY falls by 32% for every 100% increase in rent) is felt to be appropriate. This is applied to the variables. Again, these inputs were fed into *Crystal Ball* and run for 50,000 simulations.

**Figure 4a – Triangular Distribution; Likeliest, Maximum and Minimum**

**Assumptions**

**Assumption: Rent**

Triangular distribution with parameters:
- Minimum £9,500.00
- Likeliest £10,000.00
- Maximum £10,500.00

Selected range is from £9,500.00 to £10,500.00

Correlated with:
- ARY -0.32

**Assumption: ARY**

Triangular distribution with parameters:
- Minimum 4.50%
- Likeliest 5.00%
- Maximum 5.50%

Selected range is from 4.50% to 5.50%

Correlated with:
- Rent -0.32
As expected, the outcome of this simulation is to increase the output range. There is more uncertainty in the inputs, which will lead to more uncertainty in the outputs. By a comparison of the output range (or standard deviation), a client would be able to realise that the valuation of a property with vacant possession is less certain than the valuation of a corresponding property that has just been let (and hence the rent is already fixed). This is a facet that is currently overlooked in valuations that illustrates quite succinctly the importance in conveying uncertainty to a client in a way in which they can understand.

### Conclusion

As with all models, there are advantages and disadvantages to each of the distributions chosen. Similarly, each could be adjusted to better reflect market conditions at any point of time by expanding or contracting the range and varying the skewness.
There will always be debate about the appropriateness of the distribution chosen. However, for ease of use by the profession, we believe that the triangular approach is the most appropriate given the objectives.

The objective of both the Mallinson and Carsberg reports is to establish an acceptable method by which uncertainty could be expressed in a uniform and useful manner. This would require agreement on the expression of the uncertainty of the inputs and agreement on the output information that must be conveyed with each valuation.

More work will be required to agree on these issues, but the use of a Monte Carlo model, we believe is sufficiently easy, robust and accessible for the profession to consider as a possible means of expressing uncertainty in valuation.
References


RICS (2003), RICS Appraisal and Valuation Standards, Royal Institution of Chartered Surveyors, London.


